1. CLAIM: $A\cap B = B\cap A$ (I.E., $x\in A\cap B \Leftrightarrow x\in B\cap A$)

PROOF: $x\in A\cap B \overset{\text{DEF}}{\Leftrightarrow} x\in A$ AND $x\in B$
$\Leftrightarrow x\in B$ AND $x\in A$ COMMUT. / AND
$\overset{\text{DEF}}{\Leftrightarrow} x\in B\cap A$ ■

2. CLAIM: $A\cap(B\cap C) = (A\cap B)\cap C$ (I.E., $x\in A\cap(B\cap C) \Leftrightarrow x\in(A\cap B)\cap C$)

PROOF: $x\in A\cap(B\cap C) \overset{\text{DEF}}{\Leftrightarrow} x\in A$ AND $x\in B\cap C$ USE ( )! DEF OF $B\cap C$
$\Leftrightarrow x\in A$ AND $(x\in B$ AND $x\in C)$ ASSOC. / AND
$\Leftrightarrow (x\in A$ AND $x\in B)$ AND $x\in C$ DEF. OF $A\cap B$
$\Leftrightarrow x\in A\cap B$ AND $x\in C$
$\overset{\text{DEF}}{\Leftrightarrow} x\in(A\cap B)\cap C$ ■

3. CLAIM: $A\cap A = A$ (I.E., $x\in A\cap A \Leftrightarrow x\in A$)

PROOF: $x\in A\cap A \overset{\text{DEF}}{\Leftrightarrow} x\in A$ AND $x\in A$ P AND P $\Leftrightarrow$ P
$\Leftrightarrow x\in A$ ■

CLAIM: $A\cap\varnothing = \varnothing$ (I.E., $x\in A\cap\varnothing \Leftrightarrow x\in\varnothing$)

PROOF: $x\in A\cap\varnothing \overset{\text{DEF}}{\Leftrightarrow} x\in A$ AND $x\in\varnothing$ DEF. OF $\varnothing$
$\Leftrightarrow x\in A$ AND FALSE P AND FALSE $\Leftrightarrow$ FALSE
$\Leftrightarrow$ FALSE
$\Leftrightarrow x\in\varnothing$ ■ DEF. OF $\varnothing$

4. CLAIM: $A\subset B \Leftrightarrow A\cap B = A$ (I.E., $\Rightarrow$ AND $\Leftarrow$)

PROOF $\Rightarrow$: SUPPOSE $A\subset B$, I.E., $\boxed{x\in A \Rightarrow x\in B}$!

(NEED TO SHOW $\underset{?}{A\cap B = A}$, I.E., $x\in A\cap B \Leftrightarrow x\in A$
I.E., $\Rightarrow$ AND $\Leftarrow$)

- $x\in A\cap B \Rightarrow x\in A$: $x\in A\cap B \Rightarrow x\in A$ AND $x\in B$
$\Rightarrow x\in A$ ✓
- $x\in A \Rightarrow x\in A\cap B$: SUPPOSE $x\in A$.
SINCE $x\in A \Rightarrow x\in B$, WE KNOW THAT $x\in B$ TOO.
THUS $x\in A$ AND $x\in B$, SO $x\in A\cap B$ BY DEF. ✓

✓ FOR $\Rightarrow$

PROOF $\Leftarrow$: SUPPOSE THAT $A\cap B = A$, I.E., $\boxed{x\in A\cap B \Leftrightarrow x\in A}$!

(NEED TO SHOW $A\subset B$, I.E., $\underset{?}{x\in A \Rightarrow x\in B}$)

SUPPOSE $x\in A$. THEN BY HYPOTHESIS, $x\in A\cap B$,
SO BY DEFINITION, $x\in A$ AND $x\in B$
THUS, $x\in B$

✓ FOR $\Leftarrow$

WE'VE SHOWN $\Rightarrow$ AND $\Leftarrow$, I.E., $\Leftrightarrow$ ■

5. CLAIM: $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$
(I.E., $x\in A\cap(B\cup C) \Leftrightarrow x\in(A\cap B)\cup(A\cap C)$)

PROOF: $x\in A\cap(B\cup C) \overset{\text{DEF}}{\Leftrightarrow} x\in A$ AND $x\in B\cup C$ USE ( )! DEF. OF $B\cup C$
$\Leftrightarrow x\in A$ AND $(x\in B$ OR $x\in C)$ DISTRIB.
$\Leftrightarrow (x\in A$ AND $x\in B)$ OR $(x\in A$ AND $x\in C)$ DEF. OF $\cap$
$\Leftrightarrow x\in A\cap B$ OR $x\in A\cap C$ DEF. OF $\cup$
$\Leftrightarrow x\in(A\cap B)\cup(A\cap C)$ ■

6. (JUST LIKE #5, BUT WITH $\cap$ + $\cup$ INTERCHANGED)

CLAIM: $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$
(I.E., $x\in A\cup(B\cap C) \Leftrightarrow x\in(A\cup B)\cap(A\cup C)$)

PROOF: $x\in A\cup(B\cap C) \overset{\text{DEF}}{\Leftrightarrow} x\in A$ OR $x\in B\cap C$ USE ( )! DEF. OF $B\cap C$
$\Leftrightarrow x\in A$ OR $(x\in B$ AND $x\in C)$ DISTRIB.
$\Leftrightarrow (x\in A$ OR $x\in B)$ AND $(x\in A$ OR $x\in C)$ DEF. OF $\cup$
$\Leftrightarrow x\in A\cup B$ AND $x\in A\cup C$ DEF. OF $\cap$
$\Leftrightarrow x\in(A\cup B)\cap(A\cup C)$ ■