1. CLAIM: $A \setminus (A \setminus B) = A \cap B$ (I.E., $x \in A \setminus (A \setminus B) \Leftrightarrow x \in A \cap B$)

PROOF: $x \in A \setminus (A \setminus B) \overset{\text{DEF}}{\Leftrightarrow} x \in A$ AND $x \notin A \setminus B$ ↓ PULL OUT "NOT" + DEF. OF $\setminus$

$\Leftrightarrow x \in A$ AND NOT $(x \in A$ AND $x \notin B)$ ↓ NOT (P AND Q) ⟺ (NOT P) OR (NOT Q)

$\Leftrightarrow x \in A$ AND $(x \notin A$ OR $x \in B)$ ↓ DISTRIB.

$\Leftrightarrow (x \in A$ AND $x \notin A)$ OR $(x \in A$ AND $x \in B)$ ↓ P AND NOT P ⟺ FALSE

$\Leftrightarrow$ FALSE OR $(x \in A$ AND $x \in B)$ ↓ FALSE OR P ⟺ P

$\Leftrightarrow x \in A$ AND $x \in B$

$\overset{\text{DEF}}{\Leftrightarrow} x \in A \cap B$ ■

2. CLAIM: $A \setminus B = A \Leftrightarrow A \cap B = \emptyset$ (I.E., ⟹ AND ⟸)

PROOF ⟹: SUPPOSE $A \setminus B = A$ (I.E., $x \in A \setminus B \Leftrightarrow x \in A$; I.E., $\boxed{(x \in A \text{ AND } x \notin B) \Leftrightarrow x \in A}$!)

(NEED TO SHOW THAT $A \cap B = \emptyset$;
I.E., $x \in A \cap B \Leftrightarrow x \in \emptyset$;
I.E., $x \in A \cap B \Leftrightarrow$ FALSE
I.E., $\underbrace{(x \in A \text{ AND } x \in B) \text{ IS FALSE}}_{?}$)

$x \in A$ AND $x \in B$ ↘ BY HYPOTHESIS ON "$x \in A$"

$\Leftrightarrow (x \in A$ AND $x \notin B)$ AND $x \in B$ ↓ ASSOC./AND

$\Leftrightarrow x \in A$ AND $(x \notin B$ AND $x \in B)$ ↓ P AND NOT P ⟺ FALSE

$\Leftrightarrow x \in A$ AND FALSE ↓ P AND FALSE ⟺ FALSE

$\Leftrightarrow$ FALSE ✓

PROOF ⟸: SUPPOSE $A \cap B = \emptyset$ (I.E., AS ABOVE: $\boxed{(x \in A \text{ AND } x \in B) \text{ IS FALSE}}$!)

(NEED TO SHOW $A \setminus B = A$; I.E., AS ABOVE: $\underbrace{(x \in A \text{ AND } x \notin B) \Leftrightarrow x \in A}_{?}$)

$x \in A$ AND $x \notin B$

$\Leftrightarrow (x \in A$ AND $x \notin B)$ OR FALSE ↓ P ⟺ P OR FALSE

$\Leftrightarrow (x \in A$ AND $x \notin B)$ OR $(x \in A$ AND $x \in B)$ ↓ BY HYPOTHESIS

$\Leftrightarrow x \in A$ AND $(x \notin B$ OR $x \in B)$ ↓ FACTOR, I.E., ANTI-DISTRIBUTE

$\Leftrightarrow x \in A$ AND TRUE ↓ (P OR NOT P) ⟺ TRUE

$\Leftrightarrow x \in A$ ✓ ↓ P AND TRUE ⟺ P

WE'VE SHOWN ⟹ AND ⟸, I.E., ⟺ ■

3. CLAIM: $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ (I.E., $x \in \ldots \Leftrightarrow x \in \ldots$)

PROOF: $x \in A \setminus (B \cup C)$

$\overset{\text{DEF}}{\Leftrightarrow} x \in A$ AND $x \notin (B \cup C)$ ↓ DEF OF $B \cup C$

$\Leftrightarrow x \in A$ AND NOT $(x \in B$ OR $x \in C)$ ↓ NOT OF "OR"

$\Leftrightarrow x \in A$ AND $(x \notin B$ AND $x \notin C)$ ↓ "DISTRIBUTE" AND'S

$\Leftrightarrow (x \in A$ AND $x \notin B)$ AND $(x \in A$ AND $x \notin C)$ ↓ DEF OF $\setminus$

$\Leftrightarrow x \in A \setminus B$ AND $x \in A \setminus C$ ↓ DEF OF $\cap$

$\Leftrightarrow x \in (A \setminus B) \cap (A \setminus C)$ ■

4. (* JUST LIKE #3, WITH $\cap \leftrightarrow \cup$)

CLAIM: $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ (I.E., $x \in \ldots \Leftrightarrow x \in \ldots$)

PROOF: $x \in A \setminus (B \cap C)$

$\overset{\text{DEF}}{\Leftrightarrow} x \in A$ AND $x \notin B \cap C$ ↓ DEF OF $B \cap C$

$\Leftrightarrow x \in A$ AND NOT $(x \in B$ AND $x \in C)$ ↓ NOT OF "AND"

$\Leftrightarrow x \in A$ AND $(x \notin B$ OR $x \notin C)$ ↓ DISTRIBUTE

$\Leftrightarrow (x \in A$ AND $x \notin B)$ OR $(x \in A$ AND $x \notin C)$ ↓ DEF OF $\setminus$

$\Leftrightarrow x \in A \setminus B$ OR $x \in A \setminus C$ ↓ DEF OF $\cup$

$\Leftrightarrow x \in (A \setminus B) \cup (A \setminus C)$ ■

5. CLAIM: $(A \setminus B) \cup B = A \cup B$ (I.E., $x \in (A \setminus B) \cup B \Leftrightarrow x \in A \cup B$)

PROOF: $x \in (A \setminus B) \cup B$

$\overset{\text{DEF}}{\Leftrightarrow} x \in A \setminus B$ OR $x \in B$ ↓ DEF OF $\setminus$

$\Leftrightarrow (x \in A$ AND $x \notin B)$ OR $x \in B$ ↓ DISTRIBUTE

$\Leftrightarrow (x \in A$ OR $x \in B)$ AND $(x \notin B$ OR $x \in B)$ ↓ DEF OF $A \cup B$, AND (P OR NOT P) ⟺ TRUE

$\Leftrightarrow x \in A \cup B$ AND TRUE ↓ P AND TRUE ⟺ P

$\Leftrightarrow x \in A \cup B$ ■