1. CLAIM: $X \setminus \bigcup_{i\in I} X_i = \bigcap_{i\in I} (X \setminus X_i)$ (I.E., $x\in \ldots \Leftrightarrow x \in \ldots$)

PROOF:

$$
\begin{aligned}
x\in X\setminus \bigcup_{i\in I} X_i &\overset{\text{DEF}}{\Leftrightarrow} x\in X \text{ AND } x\notin \bigcup_{i\in I} X_i \\
&\Leftrightarrow x\in X \text{ AND NOT } \Big(x\in \bigcup_{i\in I} X_i\Big) \quad \text{DEF OF } \bigcup \\
&\Leftrightarrow x\in X \text{ AND NOT } (\exists\, i\in I \text{ WITH } x\in X_i) \quad \text{NOT OF } \exists \\
&\Leftrightarrow x\in X \text{ AND } (\forall\, i\in I,\ x\notin X_i) \quad \text{DISTRIBUTE} \\
&\Leftrightarrow \forall\, i\in I,\ (x\in X \text{ AND } x\notin X_i) \quad \text{DEF OF } \setminus \\
&\Leftrightarrow \forall\, i\in I,\ x\in X\setminus X_i \\
&\Leftrightarrow x\in \bigcap_{i\in I}(X\setminus X_i) \ \blacksquare \quad \text{DEF OF } \bigcap
\end{aligned}
$$

CLAIM: $X \setminus \bigcap_{i\in I} X_i = \bigcup_{i\in I} (X \setminus X_i)$ (I.E., $x\in \ldots \Leftrightarrow x \in \ldots$)

PROOF:

$$
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x\in X\setminus \bigcap_{i\in I} X_i &\overset{\text{DEF}}{\Leftrightarrow} x\in X \text{ AND } x\notin \bigcap_{i\in I} X_i \\
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&\Leftrightarrow \exists\, i\in I \text{ WITH } x\in X\setminus X_i \\
&\Leftrightarrow x\in \bigcup_{i\in I}(X\setminus X_i) \ \blacksquare \quad \text{DEF OF } \bigcup
\end{aligned}
$$

2. CLAIM: $f^{-1}\Big[\bigcup_{i\in I} X_i\Big] = \bigcup_{i\in I} f^{-1}[X_i]$ (I.E., $x\in \ldots \Leftrightarrow x \in \ldots$)

PROOF:

$$
\begin{aligned}
x\in f^{-1}\Big[\bigcup_{i\in I} X_i\Big] &\Leftrightarrow f(x)\in \bigcup_{i\in I} X_i \\
&\Leftrightarrow \exists\, i\in I \text{ WITH } f(x)\in X_i \\
&\Leftrightarrow \exists\, i\in I \text{ WITH } x\in f^{-1}[X_i] \\
&\Leftrightarrow x\in \bigcup_{i\in I} f^{-1}[X_i] \ \blacksquare
\end{aligned}
$$

CLAIM: $f^{-1}\Big[\bigcap_{i\in I} X_i\Big] = \bigcap_{i\in I} f^{-1}[X_i]$ (I.E., $x\in \ldots \Leftrightarrow x \in \ldots$)

PROOF:

$$
\begin{aligned}
x\in f^{-1}\Big[\bigcap_{i\in I} X_i\Big] &\Leftrightarrow f(x)\in \bigcap_{i\in I} X_i \\
&\Leftrightarrow \forall\, i\in I,\ f(x)\in X_i \\
&\Leftrightarrow \forall\, i\in I,\ x\in f^{-1}[X_i] \\
&\Leftrightarrow x\in \bigcap_{i\in I} f^{-1}[X_i] \ \blacksquare
\end{aligned}
$$